JOINT FLOW OF A POWDER MATRIX AND REINFORCING MESH UNDER PLANE DEFORMATION

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The high deformability of a number of the mesh structures must be taken into account when mesh-reinforced composites with a metal powder matrix are compacted by pressing or rolling. In particular, metal-weave reinforcements with a knit structure are highly deformable [1]. During joint deformation of the powder matrix and such meshes the latter manifest a certain susceptibility, especially in the early stages of compaction. In view of this it is of interest to ascertain how the introduction of a mesh affects the energy and strength parameters of the processes and, in the final account, the macrogeometry and properties of the composite. It is also important to determine the strain and force in the mesh during deformation.

Some technological processes of pressure working of the composites under discussion were analyzed in [2], where the conditions at the boundaries of a representative element (the stressed—strained state which was used to determine the stress, velocity, and density fields in the entire volume worked) place strict limitations on the scheme of technological processes, which can be described with this model. In particular, the stressed—strained state and, hence, the density as well should be macroinhomogeneous and inhomogeneity occurs only inside the representative element.

1. General Assumptions. We propose a mathematical model of a composite, the properties of whose matrix and reinforcement mesh make it possible to construct the solutions of any boundary-value problems by the methods of continuum mechanics that are applicable to the entire volume deformed. With a known mechanism of interaction between the matrix and unidirectional reinforcement elements the proposed treatment can be used to describe the deformation processes of composites with other forms of reinforcement. We consider the joint flow of a powder matrix and a metal weave much during production of sheet composites in which the mesh is much thinner than the characteristic size of a deformation center. We can thus disregard the thickness of the mesh, which can be taken to be an arbitrary reinforcing monolayer and in the calculations can be considered as the discontinuity surface of the properties of the matrix material. The solution of the corresponding boundary-value problem will also be discontinuous.

Since the real geometry of the mesh forms a cellular framework in some volume of the reinforcing monolayer, conditions are created for continuous displacement of the monolayer and matrix powder at their contact line. Such a scheme was also adopted in [3] for a mesh with a woven structure.

Henceforth we confine the discussion to an orthotropic mesh under the conditions of plane deformation ($\varepsilon_z = 0$), assuming that one of the principal directions coincides with the z axis. In this case the properties of the mesh can be determined from one diagram associating the relative elongation along the principal direction perpendicular to the z axis and the force applied in that direction. This diagram can be obtained either experimentally or theoretically [4, 5]. Since the stress—elongation relation generally should include the current porosity of the matrix and a complete set of experiments is obviously very difficult to carry out, in our view it is preferable to determine this relation theoretically. We note that the stress—strain diagram of the mesh is very nonlinear [4, 5].

The structure of the knit mesh is such that the displacements of the monolayer and the matrix at their contact line can be considered to be continuous [3]. This assumption is sufficient for building a definite system of equations, which consists of equations that describe the flow of the matrix [6], the relation between the force and the elongation of the

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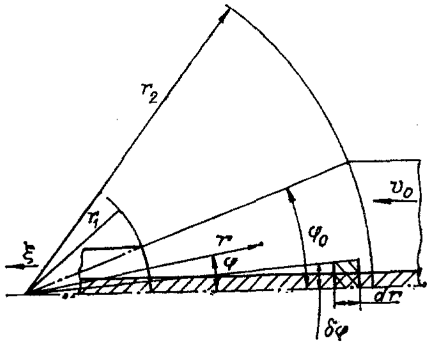

Fig. 1

mesh [5], and the condition for continuous displacements along the monolayer—matrix contact line. We note that in the general case the final strains must be considered for any model of the matrix material because of the presence of the mesh.

2. The Extrusion Process. As the operating scheme describing the compaction of the reinforcing monolayer, we study the steady-state extrusion of a viscous matrix and reinforcing monolayer under plane deformation (Fig. 1). Half of a deformation center is shown in Fig. 1 because of symmetry considerations. The surface of the tool is assumed to be ideally smooth, i.e., the frictional forces on the surface of the tool can be ignored. The flow of the powder matrix is described by the equilibrium equation, the flow law, and the continuity equation.

We introduce a cylindrical coordinate system r, ϕ, z. The solution will be sought on the basis of the following assumptions: the deformation center is delineated by the lines $r = r_1$, $r = r_2$, and the velocity $v_\phi = 0$. This assumption should be satisfied well for small angles v_ϕ, since $v_\phi = 0$ on the surface of the tool and the symmetry axis from boundary conditions. The other quantities do not depend on the polar angle ϕ.

We write the equilibrium equation on the basis of the method of Hill [7]. With the assumptions made we obtain

$$\frac{d(r\sigma_r)}{dr} - \sigma_\varphi + \frac{\tau}{\varphi_0} = 0, \tag{2.1}$$

where σ_r and σ_ϕ are the normal stresses in the matrix; and τ is the shear stress in the matrix at an arbitrary contact line. The equations of flow have the form [6]

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \left(\nu - \frac{2\mu}{3}\right)\varepsilon\delta_{ij}. \tag{2.2}$$

Here σ_{ij} are the components of the stress tensor; ε_{ij} are the components of the rate-of-strain tensor; ε is the specific rate of change in volume; ν is the bulk viscosity; μ is the shear-viscosity coefficient; and δ_{ij} is the Kronecker symbol. With the assumptions made we write the expressions for the components of the rate-of-strain tensor as $\varepsilon_r = dv/dr$, $\varepsilon_\phi = v/r$, the other components being zero ($v = v_r$ is the radial velocity). The specific rate of change in volume is $\varepsilon = dv/dr + v/r$.

Equations (2.2) can now be rewritten as

$$\sigma_r = \left(\nu + \frac{4}{3}\mu\right)\frac{dv}{dr} + \left(\nu - \frac{2\mu}{3}\right)\frac{v}{r},\ \sigma_\varphi = \left(\nu - 2\frac{\mu}{3}\right)\frac{dv}{dr} + \left(\nu + \frac{4\mu}{3}\right)\frac{v}{r}. \tag{2.3}$$

By virtue of the steady-state nature of the flow the continuity equation is

$$\frac{v}{\rho}\frac{d\rho}{dr} + \frac{dv}{dr} + \frac{v}{r} = 0 \tag{2.4}$$

(ρ is the relative density).

The relation between the force P stretching the mesh in the direction of the r axis and its relative elongation Δ from the undeformed state in the general case has the form $P = f(\Delta)$. This equation, taking into account the specific structure of the knit mesh and the properties of the wire material, was obtained numerically, e.g., in [5]. This is the form we use below.

For the compaction scheme under discussion (see Fig. 1) the relative elongation can be expressed in terms of the particle velocity. The law of motion of any point of the reinforcement layer is determined by $r = r(a, t)$ (a is the Lagrange coordination of the point and t is the time). The relative elongation then is

$$\Delta = \partial r/\partial a - 1. \tag{2.5}$$

The condition for the continuity of the velocity on the contact line implies that

$$dr/dt = v(r),$$

whence the law of motion of the point can be determined from

$$F = \int_a^r \frac{dz}{v(z)} - t = 0.$$

Differentiation of this equation gives us

$$\frac{\partial r}{\partial a} = -\frac{(\partial F/\partial a)}{(\partial F/\partial r)} = \frac{v(r)}{v(a)}.$$

Then from Eq. (2.5) we get

$$\Delta = v(r)/v(a) - 1, \tag{2.6}$$

where $v(a)$ is the velocity upon entry into the deformation center (we assume that up until then the mesh had been in the undeformed state); and $v(r)$ is the velocity of the point at a given place in the deformation center.

Taking (2.6) into account, we write the relation between the force in the mesh and the particle velocity is

$$P = f\left[\frac{v(r)}{v(a)} - 1\right]. \tag{2.7}$$

We determine the relation between the stressed state of the mesh and the matrix. With the assumptions made the equation of the virtual powers for any element dr near the symmetry axis (see Fig. 1) has the form

$$\int_r^{r+dr}\int_0^{\delta\varphi} (\sigma_r\varepsilon_r + \sigma_\varphi\varepsilon_\varphi)\, r d\varphi dr = -\int_r^{r+dr} \tau v dr - \int_0^{\delta\varphi} \sigma_r r v(r)\, d\varphi +$$
$$+\int_0^{\delta\varphi} (\sigma_r + d\sigma_r)(r + dr)\, v(r + dr)\, d\varphi.$$

Making $\delta\phi$ tend to zero, we obtain

$$\int_r^{r+dr} P\varepsilon_r dr = -\int_r^{r+dr} \tau v dr - P(r)\, v(r) + P(r + dr)\, v(r + dr)$$
$$\left(P = \int_0^{\delta\varphi} \sigma_r r\, d\varphi\right). \tag{2.8}$$

We integrate the left side of Eq. (2.8) by parts:

$$Pv|_r^{r+dr} - \int\limits_r^{r+dr} \frac{dP}{dr} v dr = - \int\limits_r^{r+dr} \tau v dr - P(r)v(r) + P(r+dr)v(r+dr).$$

In view of the arbitrary nature of v and the interval of integration over r, we find

$$\frac{dP}{dr} - \tau = 0. \tag{2.9}$$

We thus have a system of equations (2.1), (2.3), (2.4), (2.7), (2.9) in the unknowns σ_r, σ_ϕ, τ, v, ρ, and P with the boundary conditions $v = -v_0$, $\rho = \rho_0$ for $r = r_2$ and $P + \sigma_r r_1 \phi_0 = 0$ for $r = r_1$. The continuity equation (2.4) has the integral $\rho v r = -\rho_0 v_0 r_2 = c$, whence we find

$$\rho = c/(vr). \tag{2.10}$$

Substituting (2.7) into (2.9), we express τ in terms of the velocity v:

$$\tau = \frac{1}{v_0} \frac{df}{d\Delta} \frac{dv}{dr}. \tag{2.11}$$

To find v from (2.1) with allowance for (2.3), (2.10), and (2.11) we obtain the differential equation

$$r\left(\nu + \frac{4}{3}\mu\right)\frac{d^2v}{dr^2} - \frac{c}{v^2}\left(\frac{d\nu}{d\rho} + \frac{4}{3}\frac{d\mu}{d\rho}\right)\left(\frac{dv}{dr}\right)^2 - \left[\frac{2c}{rv}\left(\frac{d\nu}{d\rho} + \frac{1}{3}\frac{d\mu}{d\rho}\right) - \right.$$
$$\left. - \nu - \frac{4}{3}\mu - \frac{1}{\varphi_0 v_0}\frac{df}{d\Delta}\right]\frac{dv}{dr} - \left(\frac{d\nu}{d\rho} - \frac{2}{3}\frac{d\mu}{d\rho}\right)\frac{c}{r^2} - \left(\nu + \frac{4}{3}\mu\right)\frac{v}{r} = 0 \tag{2.12}$$

and the boundary conditions $v = -v_0$ for $r = r_2$ and

$$f\left[\frac{v(r_1)}{v_0} - 1\right] + \left[\left(\nu + \frac{4}{3}\mu\right)\frac{dv}{dr} + \left(\nu - \frac{2\mu}{3}\right)\frac{v}{r_1}\right] r_1 \varphi_0 = 0$$

for $r = r_1$. We assume the dependence of μ and ν on the relative density to be

$$\nu = \mu_0 \frac{4}{3}\frac{\rho^3}{(1-\rho)}, \quad \mu = \mu_0 \rho^2. \tag{2.13}$$

We estimate the order of the terms in (2.12). For the characteristic values we take v_0 for the velocity, r_2 for the length, r/v_0 for the time $\mu_0 r_2/v_0$ for stressses in the matrix, P_* for the force in the mesh per unit length (P_* is the force which the plastic strains of the fibers become significant), and ε_* for the strain of the mesh (ε_* is the strain in the mesh under the force $P = P_*$).

Then Eq. (2.12) can be written in dimensionless quantities as

$$r\left(\nu + \frac{4}{3}\mu\right)\frac{d^2v}{dr^2} - \frac{\rho_0}{v^2}\left(\nu' + \frac{4}{3}\mu'\right)\left(\frac{dv}{dr}\right)^2 - \left[\frac{2\rho_0}{rv}\left(\nu' + \frac{1}{3}\mu'\right) - \right.$$
$$\left. - \left(\nu + \frac{4}{3}\mu\right) - \left(\frac{P_*}{\mu_0\varphi_0 v_0 \varepsilon_*}\right)\frac{1}{v}\frac{dP}{d\Delta}\right]\frac{dv}{dr} - \left(\nu' - \frac{2}{3}\mu'\right)\frac{\rho_0}{r^2} - \left(\nu + \frac{4}{3}\mu\right)\frac{v}{r} = 0, \tag{2.14}$$
$$\nu' = \frac{d\nu}{d\rho}, \quad \mu' = \frac{d\mu}{d\rho}.$$

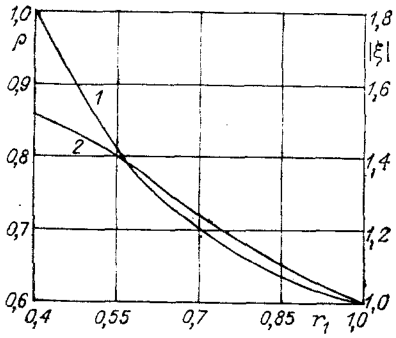

Fig. 2

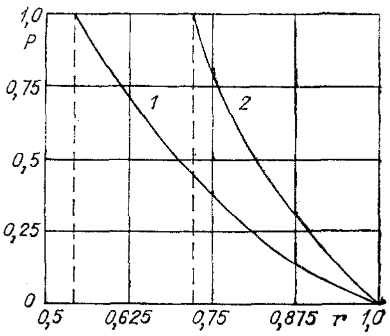

Fig. 3

The previous notation is kept for the dimensionless quantities. We have $\rho = -\rho_0/(vr)$ in the expressions for μ and ν.

From Eq. (2.14) we see that the effect of the mesh on the distribution of the particle velocity and, hence, the stressed–strained state is determined by the values of the dimensionless complex

$$\delta = \frac{P_*}{\mu_0 \varphi_0 v_0 \varepsilon_*}.$$

For an "elastic" type of mesh, made of steel and copper wires, $P_* \simeq 10^2$ N/m, $\varepsilon_* \simeq 1$ [5], the viscosity $\mu_0 \simeq 10^{10}$ Pa·sec [6], and the velocity of the piston during extrusion is $v_0 \simeq 10^{-2}$ m/sec [9]. Thus, for the process under consideration we have $\delta \simeq 10^{-6}$ for $\phi_0 \simeq 1$. From this we see that the introduction of pliable metal-weave meshes does not significantly affect the distribution of the velocities and, hence, the stressed–strained state and density in the matrix. The processes of deformation of the matrix and mesh can thus be considered separately: first we determine the fields of all the physical quantities from the solution of the problem of deformation of the powder matrix and then we use (2.7) with the determined velocity field to calculate the force in the mesh. Equation (2.14) becomes much simpler when the term due to the presence of the mesh is discarded.

We introduce the notation $u = vr$. Then, to determine u we obtain from (2.14) the expression

$$a_1 r^2 \frac{d^2u}{dr^2} - \rho_0 a_1' r^2 \left(\frac{du}{dr}\right)^2 - \left(a_1 - \frac{2\rho_0 \mu'}{u}\right) r \frac{du}{dr} = 0,$$

where $a_1 = \nu + \frac{4}{3}\mu$; $a_1' = \frac{da_1}{d\rho}$; $\mu = \mu(u)$; and $\nu = \nu(u)$. Substituting $z_1 = \ln r$ and $u = u(z_1)$,

$$a_1 \frac{d^2u}{dz_1^2} - \rho_0 a_1' \left(\frac{du}{dz_1}\right)^2 - 2\left(a_1 - \frac{\rho_0 \mu'}{u}\right) \frac{du}{dz_1} = 0.$$

We take u to be an independent variable, whereupon the last equation becomes

$$\frac{d^2 z_1}{du^2} + \frac{\rho_0 a_1'}{a_1} \frac{dz_1}{du} + 2\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right)\left(\frac{dz_1}{du}\right)^2 = 0.$$

Setting $dz_1/du = y$, we arrive at the Riccati equation

$$\frac{dy}{du} + \frac{\rho_0 a_1'}{a_1} y + 2\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right) y^2 = 0,$$

which the substitution $y = x^{-1}$ reduces to the linear equation

$$\frac{dx}{du} - \frac{\rho_0 a_1'}{a_1} x - 2\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right) = 0. \tag{2.15}$$

We determine z_1 from the equation

$$\frac{dz_1}{du} = x^{-1}. \tag{2.16}$$

The solution of the system (2.15), (2.16) can be written in quadratures. The boundary conditions have the form $z_1 = 0$ for $u = -1$ and $x = (a_1 - b)\xi/a_1$ for $u = \xi$ (ξ is the value of the function u at the exit from the deformation center, $b = \nu - 2\mu/3$).

The solution of Eqs. (2.15) and (21.6) is written as

$$x = \exp(-F)\left[\left(\frac{a_1 - b}{a_1}\right)\xi + 2\int_{\xi}^{u}\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right)\exp(F)\,du\right],$$

$$F = -\rho_0 \int_{\xi}^{u} \frac{d(\ln a_1)}{d\rho}\,du,$$

$$z_1 = \int_{-1}^{u} \exp(F)\left[\left(\frac{a_1 - b}{a_1}\right)\xi + 2\int_{\xi}^{u}\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right)\exp(F)\right]^{-1} du.$$

We find ξ from

$$\ln r_1 = \int_{-1}^{\xi} \exp(F)\left[\left(\frac{a_1 - b}{a_1}\right)\Big|_{u=\xi}\xi + 2\int_{\xi}^{u}\left(1 - \frac{\rho_0 \mu'}{a_1 u}\right)\exp(F)\,du\right]^{-1} du.$$

If μ and ν are determined by Eq. (2.13), then

$$F = \frac{(\xi^2 - u^2)}{2} + \rho_0^2 \ln\left(\frac{\rho_0 - \xi}{\rho_0 - u}\right) + \frac{(\rho_0 + \xi)^2 - (\rho_0 + u)^2}{2},$$

$$\ln r = z_1 = \int_{-1}^{u} \exp(F)\left[\frac{3}{2}(\xi - \rho_0) + \int_{\xi}^{u}\left(\frac{3\rho_0}{u} - 1\right)\exp(F)\,du\right]^{-1} du,$$

$$\ln r_1 = \int_{-1}^{\xi} \exp(F)\left[\frac{3}{2}(\xi - \rho_0) + \int_{\xi}^{u}\left(\frac{3\rho_0}{u} - 1\right)\exp(F)\,du\right]^{-1} du.$$

The results of calculations for this case are given in Fig. 2 (dependence of the velocity of outflow of the material and the density of the product on r_1 (curves 1, 2) and the force in the mesh along the deformation center for various parameters of the "elastic" weave mesh (curve 3). This type of weave is characterized by the parameter θ_0 [5]. In Fig. 3 curves 1, 2 correspond to $\theta_0 = 1.484$ and 0.904. These graphs can be used to determine the critical deformation center size r_* at which the material of the mesh undergoes substantial plastic deformations, thus degrading the properties of the product. In particular, $r_* = 0.55$ for $\theta_0 = 1.484$ and $r_* = 0.72$ for $\theta_0 = 0.904$.

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CALCULATION OF THE RESIDUAL STRESSES IN WELDED JOINTS OF HARD ALLOYS WITH STEELS BY THE BOUNDARY ELEMENTS METHOD

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We have used the boundary elements method to study the pattern of the residual-stress distribution in welded cylindrical specimens of a hard alloy and steel. The experimentally observed expansion of steel as a result of internal transformations is prescribed by uniform bulk deformation. It has been proved theoretically and experimentally that the concentration of the axial tensile stresses on the cylindrical surface in the zone of the welded joint causes the hard alloy to fracture. It has also been proved that controlling the cooling rate not only reduces the residual stresses and prevents fracturing of the hard alloy but also forms a residual capable of compensating, to a degree, for the working stresses in the welded member.

Mechanism of Residual-Stress Formation. Hard alloys are used in industry to fit out boring, cutting, stamping, and other tools. Permanent connections are made by welding, brazing, anf gluing. Welded joints are strongest. The existence of residual stresses as an inevitable consequence the thermal conditions of the welding, however, weakens the welded joint and may cause the hard alloy to fracture.

The mechanism of stress formation in welded joints is associated with the cooling process and is due to the difference in the thermal expansion coefficients of the materials being welded. Reduction of the residual stresses is promised by using various compensating metal spacers and powder interlayers and by artificially producing porosity in the zone of the joint [1]. As this analysis shows, it is more promising to control the stressed state by means of the volume expansion effect that accompanies the structural transformations of the steel during cooling.

The main laws of stress formation during welding of diverse materials can be traced on the simplest model of uniaxial strain. During cooling in the free state the thermal strains of the steel and the hard alloy are $\alpha_1\Delta T$ and $\alpha_2\Delta T$, respectively (ΔT is the temperature drop and α_1 and α_2 are the thermal expansion coefficients (TECs) of the steel and the alloy). If ε_0 is the structural deformation of the steel and ε_1 and ε_2 are the residual strains of the steel and the alloy, then the condition for a rigid joint is

$$\varepsilon_2 + \alpha_2\Delta T = \varepsilon_0 + \varepsilon_1 + \alpha_1\Delta T.$$

From this we determine the residual strain of the hard alloy

$$\varepsilon_2 = \varepsilon_1 + \varepsilon_0 - (\alpha_2 - \alpha_1)\Delta T.$$

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